Fermi National Accelerator Laboratory

FERMILAB-Pub-86/61-T
July 3, 1986

Nonabelian Family Symmetry and the Origin of Fermion Masses and Mixing Angles

Mark Soldate
M. Hall Reno
Christopher T. Hill

Fermi National Acceleratory Laboratory
P.O. Box 500
Batavia, IL 60510

We study the origin of fermion masses and mixing angles in a class of gauged family symmetry models broken by elementary Higgs scalars at $\approx 10^3$ TeV. We find that large hierarchies among fermion masses can be produced more naturally in a model with four generations rather than three.

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

I. Introduction

The general principle of gauge invariance has led to a unified understanding of most of the basic forces in nature. It has suggested specific unified field theories of the known, and possibly new, interactions. The Standard Model has also led, however, to a set of "Standard Puzzles", including the problem of fermion masses and mixings. The specific fields responsible for the elementary fermion masses and mixing angles are conventionally described in terms of elementary scalar particles and Yukawa interactions. Although it is conceivable that scalar interactions are remnants of some gauge structure near the weak scale, i.e. technicolor[1] and a requisite extension[2], no compelling model of this sort has yet been given. This suggests that the scalar interactions may be viewed as fundamental over a large range of energies above M_W and extending upward for several orders of magnitude. Thus we have, in the problem of fermion masses and mixing angles, either a fundamental departure from the principle that all interactions involve gauge vector bosons, or that the relevant gauge interactions here are filtered through various dynamical effects over a large range of energies. It is not clear that the principle of gauge invariance alone is a reliable guide to understanding the persistent problem of fermion masses and mixing angles. This set of issues may be a harbinger of a great deal of structure to come immediately beyond the electroweak scale. Perhaps the grand desert is in fact a rainforest.

Our objective in the present letter is to pursue an understanding of quark mass matrices, mixing angles and generational hierarchy problems within the context of $SU(3) \times SU(2) \times U(1)$ and a nonabelian family group[3]. We are not particularly mindful of unification constraints, yet we hope for some simplicity and will allow only one Higgs–Yukawa coupling constant per charge species. Our principal aim is to achieve a generation hierarchy without essentially putting one in by hand and without much fine–tuning, even though the price paid is the introduction of more Higgs structure.

We shall consider both $SU(3)_F$ and $SU(4)_F$ gauged family groups and sketch only the general aspects. The $SU(3)_F$ model is essentially a simple first start, but leads to fine–tuning problems that are relaxed though do not completely disappear in the $SU(4)_F$ case. Successive vacuum expectation values (*vev*'s) completely break the family symmetry, creating a mass hierarchy based on ratios of *vev*'s to the fourth power. In this way, small splittings between family symmetry breaking scales are magnified to account for large fermion mass splittings. The Cabibbo angle is qualitatively correct, depending on $((m_u/m_c)^{1/4}, (m_d/m_s)^{1/4})$. By the same token, though, the long lifetime of the b must be accounted for by fortuitous cancellations or accidentally small parameters. While in principle CP violating phases appear, we do not address the question of the origin of CP violation here. Our models rely on an extended Higgs structure that in general has flavor-changing neutral current (FCNC) couplings. Several mechanisms suppress these FCNC effects. The full details of these models will be presented elsewhere[4].

II. Mass Matrices in the SU$(3)_F$ Model

The natural choice for the transformation of $SU(2)_L$ doublet quarks is the fundamental representation of $SU(3)_F$. With the requirement of one Yukawa coupling per charge species, we are led to consider weak singlet quarks transforming as triplets or anti-triplets. Only the latter choice is consistent with a mass matrix with no near degeneracies, barring delicate cancellations. Therefore, we choose the Higgs doublet h to belong to the two-indexed symmetric representation, 6_F[5]. (The choice of complex conjugate representations for both quarks and scalars is equivalent.) Because weak singlet up- and down-type quarks have different weak hypercharges, it is necessary to introduce two multiplets, h_U and h_D, with hypercharges $+1/2$ and $-1/2$, respectively. The quark Higgs-multiplet couplings are then

$$g_U\ \bar{Q}_{Lj}\ U_{Rk}\ h_U^{jk}\ +\ g_D\ \bar{Q}_{Lj}\ D_{Rk}\ h_D^{jk} \tag{2.1}$$

where Q_L, U_R, and D_R represent $SU(2)_L$ doublets, charge $2/3$ singlets, and charge -1/3 singlets. We reserve a few comments on leptons for later.

The family group will be gauged in order to minimize the hierarchy of mass scales needed to satisfy FCNC bounds. The gauge interactions play no role in the generation of fermion masses, and receive little discussion until the analysis of FCNC effects. Spectator fermions with $SU(3)_F$ quantum numbers alone must be introduced to cancel the $SU(3)_F$ gauge anomalies associated with the quark (and lepton) representations. A few of their properties are briefly discussed later.

The family symmetry breaking scale must be higher than the weak scale($\sim 10^3$ TeV)[6] if large FCNC effects are to be avoided. To accomplish the splitting of weak and family scales, we introduce $SU(3) \times SU(2) \times U(1)$ singlet scalar multiplets in the fundamental representation of $SU(3)_F$, denoted generically as Φ_a^j, $a = 1, 2$. The breaking of the family group by Φ_a will split the Higgs multiplets in a way consistent with FCNC constraints, as well as give mass to the family gauge bosons.

We begin by examining the most general Higgs potential involving Φ_a, $a = 1, 2$ and h_Q, $Q = U, D$. The potential involving Φ_a alone is considered first, with the mixed $\Phi_a - h_Q$ couplings treated as perturbations. With the Φ_a fields shifted to their vacuum values, we analyze the h-sector, making the fine-tunings necessary to arrange for weak symmetry breaking. The pattern of vev's of h_Q is proportional to the quark mass matrix.

Assuming that $|\langle\Phi_1\rangle|^2 > |\langle\Phi_2\rangle|^2 > 0$, an $SU(3)_F$, then an $SU(2)_F \subset SU(3)_F$ rotation can be performed to set:

$$\langle\Phi_1\rangle = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle\Phi_2\rangle = \begin{pmatrix} v_1' \\ v_2 \\ 0 \end{pmatrix}, \tag{2.2}$$

with $|v_1| > |v_2, v_1'| \geq 0$. We may always engineer the "antiferromagnetic case," $v_1' = 0$, which we shall presently assume; the qualitative features of our results are unchanged by allowing nonzero v_1'.

There are three types of dimension-four interactions of Φ_a with h_Q. The first kind are those that do not distinguish between the family quantum numbers of h_Q, so these interactions shift the overall scale of the h-masses by a constant. We must make two fine-tunings so that the masses-squared of h_Q, $-\mu_Q^2$, is negative, and much less than v_1^2 or v_2^2, on the order of the weak scale.

Interactions of the form

$$A_{ab}^U \, \Phi_a^j \, \Phi_{bk} \, h_{Ujl} \, h_U^{lk} \; + \; (U \to D) \tag{2.3}$$

give rise to mass contributions to the h-multiplets that distinguish between different components. With $\langle\Phi\rangle \neq 0$, all h_Q get masses on the order of v_1 or v_2 except h_Q^{33}[7]. With only two light scalar doublets, FCNC constraints can be satisfied.

There is a residual global invariance G_0 respected by the Lagrangian with these two types of interactions for h with Φ. The third type of interaction does not preserve G_0. The symmetry G_0 is a combination of $SU(3)_F$, a $U(1)$ in the h_Q and quark sectors, an a $U(1)$ in the Φ sector. The global symmetry allows quark masses only for the third generation and no mixing.

The third type of $\Phi - h$ interaction contains $\epsilon-$tensors. The unique dimension-4 interactions formed with the given fields and $\epsilon-$tensors are:

$$X_{ab}\ \Phi^j_a\ \Phi^{j'}{}_b\ h_U^{kk'}\ h_D^{ll'}\ \epsilon_{jkl}\ \epsilon_{j'k'l'}\ +\ \text{h.c.} \tag{2.4}$$

where for economy $X_{21} = 0$. These couplings break G_0 to a $\mathbf{Z}_2$ subgroup. The $\mathbf{Z}_2$ symmetry allows the first two quark generations to get masses but it forbids the mixing of the third generation with the other two. We return to the problem of third generation mixing below.

The quark mass matrices are determined by the parameters in the Higgs potential exhibited in (2.3) and (2.4). We assume that the parameters specifying any one set of interactions are of about the same size e.g. $A_{U11} \simeq A_{U12} \simeq A_{U22}$, and expand quantities in the ratio $\epsilon_{12} = v_2/v_1, |\epsilon_{12}| \leq 1$. In principle we should diagonalize the full scalar mass matrix, and write an effective Lagrangian in terms of the light scalars composed primarily of h_Q^{33}. The light scalar vev's break $SU(2)_L$ and generate quark masses. In practice, the task is simplified for two reasons: the ratio m_s/m_b requires the Higgs potential parameters to obey $|X_{ab}| \ll 1$, and large portions of the scalar mass matrix are decoupled. The qualitative features of the quark mass matrices are obtained by the diagonalization of the scalar mass matrices arising from (2.3), then perturbatively treating the interactions in (2.4). We proceed to describe the results.

The mass matrix written as

$$(M_U^2)^i_j\ =\ A^U_{ab}\ < \Phi^i_a > < \Phi_{bj} > \tag{2.5}$$

is easily diagonalized. Expanding in terms of the ratio ϵ_{12}, we find that M_Q^2 in

$$(M_Q^2)^i_j\ h_{Qik}\ h_Q^{jk}\ \ \rightarrow\ \ (D_Q^2)^i_j\ h'_{Qik}\ h'^{jk}_Q \tag{2.6}$$

is diagonalized by the rotation

$$R_Q \simeq \begin{pmatrix} 1 & -\epsilon_Q & 0 \\ \epsilon_Q & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.7}$$

where $\epsilon_Q = (A_{Q12}/A_{Q11})\ \epsilon_{12}$. Here, adding in the overall $-\mu_Q^2$ to the scalar masses

$$D_Q^2 - \mu_Q^2 \simeq diag\{A_{Q11}v_1^2 - \mu_Q^2, A_{Q11}v_1^2(a_Q - 1)\epsilon_Q^2 - \mu_Q^2, -\mu_Q^2\}, \tag{2.8}$$

with $a_Q = A_{Q11}A_{Q22}/A_{Q12}^2$ assumed to be greater than 1. Since μ_Q^2 is tuned for electro–weak breaking, its only appreciable effect is to give mass to h_Q^{33}.

The rotations, R_U and R_D, together with the first order perturbative treatment of the induced tadpoles from (2.4) generate the quark mass matrices. The tadpoles feed down the vev's of h_Q^{33} into the $h_Q'^{ij},\ i,j = 1,2$ sector. The Yukawa couplings of the h' scalars are

$$g_U(\bar{Q}_L R_U)_j U'_{Rk} h'^{jk}_U + g_D(\bar{Q}_L R_D)_j D'_{Rk} h'^{jk}_D. \tag{2.9}$$

The $SU(2)_L$ singlet quarks have already been rotated by R_Q, since there are no charged currents with singlet quarks. Explicitly, the partially rotated up–type quark mass matrix (which is symmetric) is

$$\mathcal{M}'_U = g_U \ < h'^{jk}_U > \tag{2.10}$$

$$\simeq g_U \begin{pmatrix} v_d(X_{11}\epsilon_U^2 - X_{12}\epsilon_{12}\epsilon_U + X_{22}\epsilon_{12}^2)/A_{U11} & \cdot & \cdot \\ v_d(2X_{11}\epsilon_U - X_{12}\epsilon_{12})/A_{U11} & v_d X_{11}/((a_U - 1)\epsilon_U^2 A_{U11}) & \cdot \\ 0 & 0 & v_u \end{pmatrix}$$

and similarly for $< h'^{jk}_D >$. The particular pattern in ϵ_{12} arises because, for example, h'^{11}_U with mass $\sim v_1$ receives a tadpole $\sim v_2^2 v_d$, while h'^{22}_U with mass $\sim v_2$ receives a tadpole $\sim v_1^2 v_d$.

By assumption the X_{ij} are all of the same order so that the ratio of the first and second generation masses is dominated by their relative dependence on ϵ_U or ϵ_D. Extracting this dependence gives

$$\begin{aligned} (m_u, m_c, m_t) &\simeq g_U(Y_u\,\epsilon_U^2 v_d,\ Y_c\,v_d/\epsilon_U^2,\ v_u), \\ (m_d, m_s, m_b) &\simeq g_D(Y_d\,\epsilon_D^2 v_u,\ Y_s\,v_u/\epsilon_D^2,\ v_d), \end{aligned} \tag{2.11}$$

where Y_q is a function of X_{ij} and A_{Qab}. Roughly, $m_u/m_c \sim \epsilon_U^4$ and $m_d/m_s \sim \epsilon_D^4$. The unusual fact that these ratios depend on ϵ_{12} to the fourth (not second) power has the desireable feature that the ratio ϵ_{12} does not have to be especially small; $\epsilon_{12} \simeq 1/3$ will be assumed hereafter. The matrix entering eq.(2.10) is nearly diagonal, so to a good approximation, the K–M matrix is $K \simeq R_U\, R_D^T$. Corrections are of order ϵ_{12}^3 or smaller. The unconventional approximate relation between the ratios of first and second generation fermion masses to the expansion parameter ϵ_{12} means that the Cabibbo angle depends approximately on the fourth roots of the ratios of light quark masses. In the limit of $Y_d = Y_s$ and $Y_u = Y_c$ in eq.(2.11), the Cabibbo angle is

$$\theta_{Cab} \simeq (m_d/m_s)^{\frac{1}{4}} - (m_u/m_c)^{\frac{1}{4}}. \tag{2.12}$$

For quark masses $m_d = 10$ MeV, $m_u = 5$ MeV, $m_s = 150$ MeV, and $m_c = 1.2$ GeV, this relation is comparable to the often quoted result[8] $\theta_{Cab} \simeq (m_d/m_s)^{\frac{1}{2}}$. There are too many parameters, however, to make a precise statement about the size of the Cabibbo angle.

The ratio of m_s/m_b may be used to estimate X,

$$\frac{m_s}{m_b} \simeq \frac{v_u}{v_d}\frac{v_1^2}{m_{h'_{D22}}^2}|X|. \tag{2.13}$$

Assuming $v_u \simeq v_d$, $|X| \lesssim 10^{-2}$, an unsatisfactorily small value for a tree-level parameter. One of the reasons for our preference of $SU(4)_F$ over $SU(3)_F$ is that the parameters corresponding to X in the $SU(4)_F$ case must be generated radiatively and are therefore naturally small.

Third generation mixing arises through effects generated by additional fields. One appealing approach is to introduce a third $SU(3)\times SU(2)\times U(1)$ singlet scalar multiplet, Φ_3. With appropriate relabelings of Φ_a, it can be assumed that the *vev*'s of Φ_1 and Φ_2 are specified by eq.(2.2), and the *vev* of Φ_3 is

$$<\Phi_3> = \begin{pmatrix} v_1' \\ v_2'' \\ v_3 \end{pmatrix}, \tag{2.14}$$

with $|v_1| \geq |v_2| \geq |v_3|$. The *vev* v_3 breaks the $\mathbf{Z}_2$ symmetry, so now the third generation mixes with the first two. We will carry out expansions in ϵ_{12} and $\epsilon_{23} = v_3/v_2$.

Dropping the subscript U or D, the matrix analogous to that of eq.(2.7) is

$$R \simeq \begin{pmatrix} 1 & -a_{12}\epsilon_{12} & b_{13}\epsilon_{12}\epsilon_{23} \\ a_{12}\epsilon_{12} & 1 & -b_{23}\epsilon_{23} \\ a_{13}\epsilon_{12}\epsilon_{23} & b_{23}\epsilon_{23} & 1 \end{pmatrix} \tag{2.15}$$

where $a_{jk} = A_{jk}/A_{11}$, $b_{23} = (a_{23} - a_{12}a_{13})/(a_{22} - a_{12}^2)$, $b_{13} = b_{23}a_{12} - a_{13}$. For convenience, v_1'' and v_2' have been set to zero. Setting $\epsilon_{23} \simeq 1/40$ yields an appropriate mixing between the second and third generations. To leading order, the ratio m_s/m_b is independent of ϵ_{23}, so unlike the Fritzsch case, θ_{bc} cannot be related to quark mass ratios. Third family mixing with the first is even smaller than θ_{bc}.

The diagonalized mass matrices analogous to eq.(2.8) are

$$D^2 \simeq A_1 v_1^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & (a_{22} - a_{12}^2){\epsilon_{12}}^2 & 0 \\ 0 & 0 & [a_{33} - a_{13}^2 - b_{23}^2(a_{22} - a_{12}^2)]{\epsilon_{12}}^2{\epsilon_{23}}^2 \end{pmatrix}, \tag{2.16}$$

dropping subscripts U or D again. We shall see in section 4 that FCNC effects require $A_1 v_1^2 \gtrsim (170 \text{ TeV})^2$. The form for D^2 makes it clear that h_Q^{33} is much lighter than the other members of their respective multiplets because ϵ_{23}^2 is small, but it may be necessary to adjust μ_Q^2 to keep the light scalar masses below 1 TeV.

The masses and mixing angles in this $SU(3)_F$ model with three Φ's can be satisfactorily chosen, but the small values of the X's in (2.4) are rather artificial. The motivation for the study of $SU(4)_F$ is to attempt to avoid this difficulty.

III. The $SU(4)_F$ Model

The $SU(4)_F$ extension of the model presented in the last section yields analogous results. Aside from the obvious extra generation, the main differences lie in the $\Phi - h$ interactions. Interactions of (2.3) are the same in the $SU(4)_F$ case, but the G_0 violating terms corresponding to (2.4) are dimension-6. There are six distinct interactions of this type which can be written in the form

$$\frac{X_{abcd}}{M^2}\,\Phi^j_a\,\Phi^{j'}_b\,\Phi^k_c\,\Phi^{k'}_d\,h^{ll'}_U\,h^{mm'}_D\,\epsilon_{jklm}\,\epsilon_{j'k'l'm'}+\text{h.c.} \tag{3.1}$$

We expect such terms are radiatively induced, with M of order the $SU(4)_F$ breaking scale, and thus naturally small; (more scalars in addition to Φ_a and h_Q must be introduced to generate these effective operators[4]).

The $SU(3)\times SU(2)\times U(1)$ singlet fields $\Phi_a,\ a=1,2,3$ can be assumed to take *vev*'s

$$<\Phi_1>=\begin{pmatrix} v_1\\ 0\\ 0\\ 0\end{pmatrix},\ <\Phi_2>=\begin{pmatrix} v_1'\\ v_2\\ 0\\ 0\end{pmatrix},\ <\Phi_3>=\begin{pmatrix} v_1''\\ v_2'\\ v_3\\ 0\end{pmatrix}, \tag{3.2}$$

where $|v_1|>|v_2|>|v_3|>0$.

The structure of the three family sector is identical to the $SU(3)_F$ model with three triplet scalars. Now, h^{44}_Q enjoy special status as the light scalars. The *vev*'s of the symmetric h'_D are

$$<h'_D>\simeq\begin{pmatrix} \eta_{D11}\epsilon^4_{12}\epsilon^4_{23}v_u & \cdot & \cdot & \cdot\\ \eta_{D12}\epsilon^3_{12}\epsilon^4_{23}v_u & \eta_{D22}\epsilon^4_{23}v_u & \cdot & \cdot\\ \eta_{D13}\epsilon^3_{12}\epsilon^3_{23}v_u & \eta_{D23}\epsilon^3_{23}v_u & \eta_{D33}v_u & \cdot\\ 0 & 0 & 0 & v_d\end{pmatrix}. \tag{3.3}$$

The η_D's are functions of X's and A's, the details of which are not particularly relevant for our discussion here.

Our conclusions about the Cabibbo angle and the ratios of m_u/m_c and m_d/m_s are qualitatively unchanged by the transition from $SU(3)_F$ to $SU(4)_F$. The mass ratios scale as ϵ^4_{12}. Now, m_s/m_b and m_c/m_t are of order ϵ^4_{23}, suggesting that ϵ_{23} is not particularly small, $\epsilon_{23}\simeq 2/5$.

In the absense of other scalar interactions, the dominant contribution to the K–M matrix K is again $R_U R^T_D$ where R_Q is given by eq.(2.15). Evident from eq.(3.3), additional rotations are small compared to R_Q. Third generation mixing is typically too large, but cancellations between R_U and R^T_D, or small values for coefficients such as b_{23} of eq.(2.15) could lead to acceptable values.

Fourth generation mixing can arise only after the addition of more fields. A field Φ_4 could be added, but it would suffer the same fine tunings called for by Φ_3

in $SU(3)_F$. An alternative is to add heavy fields which radiatively induce effective operators of the type

$$\frac{Z_{Qa}}{M^2}\,\Phi^j_a\,h_{Qjk}\,h^{kl}_Q\,\epsilon_{lmns}\,\Phi^m_1\,\Phi^n_2\,\Phi^s_3\;+\;\text{h.c.} \tag{3.4}$$

A specific example may be found in ref. [4]. These effective operators give

$$\frac{< h'^{43}_Q >}{< h'^{33}_Q >} \sim (\frac{Z_Q}{X})\epsilon_{12}\epsilon^2_{23}. \tag{3.5}$$

The origin of the suppression factors proportional to ϵ_{23} is the subgroup $SU(2)_F$ which in the limit $v_3 \rightarrow 0$ demands that the tadpole of h'^{43} vanish while the tadpole of h'^{33} does not. Assuming the factor in parentheses is of order one, the mixing between b' and b is $\sim (m_b/m_{b'})\epsilon_{12}\epsilon^2_{23} \lesssim 10^{-2}$. This suggests that the fourth generation is weakly mixed, $\theta_{b't} \lesssim 10^{-2}$. Assuming $m'_b \simeq 100$ GeV, a typical lifetime is of order 10^{-18} sec. With $\theta_{b't}$ so small, the K–M matrix of the first three generations is little affected by fourth generation mixing, as assumed previously.

The ratio of $m_b/m_{b'}$ can be used to estimate the size in $SU(4)_F$ of the X's again assumed to be of roughly the same magnitude.

$$\frac{m_b}{m_{b'}} \simeq (\frac{v_1^2 v_2^2}{M^2 D^2_{33}})\frac{v_u}{v_d}|X| \tag{3.6}$$

where D^2_{33} is given in eq.(2.16). Taking $M^2 \simeq v_1^2$, $m_b/m_{b'} \lesssim 1/5$, and $v_u \simeq v_d$ gives $|X| \lesssim 1/30$. This is a significant improvement over the $SU(3)_F$ model, in that the small values of the X's can be understood as a radiative effect in the $SU(4)_F$ model.

IV. General considerations and conclusions

Bounds on the flavor breaking scale come from low energy measurements of flavor violating processes, notably the $K_L - K_S$ mass difference, ΔM_K. Family gauge bosons, as well as neutral Higgs scalars can mediate $\Delta S = 2$ processes at tree level. There are charged scalar contributions to ΔM_K at one loop, but they are automatically suppressed if the tree level constraints are satisfied. We shall calculate the tree level contributions to ΔM_K, requiring the gauge boson and scalar effects each separately to be less than the observed value in magnitude.

The terms in the effective $\Delta S = 2$ Hamiltonian due to family gauge boson exchange can be written as a sum of four-quark operators $\mathcal{O}_i$ with coefficients c_i:

$$(\mathcal{H}_{\rm eff}^{\Delta S=2})_{gauge\ boson} = \sum_i c_i\ \mathcal{O}_i. \tag{4.1}$$

One such operator is $(\bar{s}_L\gamma^\mu d_L)(\bar{s}_R\gamma_\mu d_R)$ with coefficient approximately $-(1/v_1^2 + \epsilon_D^2/(2v_2^2))$. Of the other terms, some have coefficients suppressed by rotation angles, others have operator structure which in many models[9] leads to smaller matrix elements. As a very rough guide, we use the vacuum insertion approximation to estimate the matrix element of the four-quark operators. Setting the magnitudes of these contributions to ΔM_K to be less than the observed value yields

$$(\frac{1}{v_1^2} + \frac{\epsilon_D^2}{2v_2^2})^{-1} \gtrsim (4 \times 10^3\ \mathrm{TeV})^2. \tag{4.2}$$

If we take $\epsilon_D = 1/2$, the bound is satisfied by

$$v_1 \simeq 3\,v_2 \quad \gtrsim \quad 6 \times 10^3\ \mathrm{TeV}. \tag{4.3}$$

The gauge boson contribution to ΔM_K has the same sign as the experimental result. If the family symmetry had not been gauged, the existence of familons arising from the spontaneous breaking of the symmetry would have put a significantly more stringent bound on v_1[7].

The contribution to ΔM_K from a complex scalar with interaction $g_{eff}H(\bar{d}_L s_R + \bar{s}_L d_R)$ + h.c. is

$$< K^0|\mathcal{H}_{\rm eff}^{\Delta S=2}|\bar{K}^0 >_{scalar} \simeq -\frac{g_{eff}^2}{2m_H^2} < K^0|[(\bar{s}_L d_R)(\bar{s}_R d_L) + (L \leftrightarrow R)]|\bar{K}^0 > . \tag{4.4}$$

Again using the vacuum approximation to the matrix element,

$$\frac{m_H}{g_{eff}} \gtrsim 4.3\ \times 10^3\ \mathrm{TeV}. \tag{4.5}$$

The mass shift has the opposite sign relative to the gauge boson contribution. We proceed to describe the calculation of the effective couplings and corresponding mass limits for the various scalars in the $SU(3)_F$, and briefly $SU(4)_F$, theories.

For simplicity, we begin with the $SU(3)_F$ model. The calculation of the FCNC effects of the scalars should be carried out in a mass eigenstate basis. This requires that the mixings of h'_Q from the interactions of (2.4) be diagonalized. There are two classes of scalars: those with masses-squared proportional to v_1^2 or v_2^2 (we call these "heavy"), and the two "light" Weinberg–Salam doublets. We begin with the second set. Before including the third triplet Φ_3, there is a natural Glashow–Weinberg–Paschos[10] mechanism of flavor conservation by the light scalars. Only one light Higgs scalar couples to the d–s sector, so its Yukawa couplings are diagonalized with the quark mass matrix. A different light scalar gives mass to the b quark, but the $\mathbf{Z}_2$ symmetry protects against flavor violation due to scalar mixing. When the third triplet is incorporated to make the model realistic, the two light complex scalars mix. We estimate the mixing to be on the order of

$$m^2\ h_U'^{33}\ h_D'^{33}\ \sim\ X\ v_1^2\ \theta_{bc}^2\ h_U'^{33}\ h_D'^{33}. \tag{4.6}$$

This may be large: $Xv_1^2\theta_{bc}^2 \sim (750\ \mathrm{GeV})^2$, indicating that the light mass eigenstates may have masses close to their unitarity bound. Their effect on ΔM_K is small, however, as $g_{eff} \simeq \theta_{bs}\,\theta_{bd}\,g_D$, proportional to third generation mixing angles. Mixing of $h_U'^{33}$ and $h_D'^{33}$ in the absence of third generation fermion mixing does not lead to FCNC effects because the diagonalization of the quark mass matrices still diagonalizes the couplings of the light scalar mass eigenstates to the quark mass eigenstates.

Although some of the heavy scalars have suppressed strangeness–changing Yukawa's, the mass eigenstate related to h_D^{12} has a full strength coupling, $g_{eff} \sim g_D = m_b/v_d \simeq 0.03$. The mass bound on it is approximately 170 TeV. The lightest of the heavy scalars have mass-squared $\sim v_2^2$; the mass eigenstate related to h_D^{22} has the largest g_{eff}. It is approximately $\epsilon_D^3 g_D$, arising from the rotation required to diagonalize M'_D of eq.(2.10). From ΔM_K, this scalar's mass must be greater than $\epsilon_D^3 \cdot 170$ TeV. However, its mass has already been indirectly bounded by $\epsilon_D \cdot 170$ TeV following from the first bound and the pattern of scalar masses, so this second direct bound is already satisfied.

The constraints on the $SU(4)_F$ Higgs scalars are rather similar. The effective FCNC Yukawa's of the light scalars are suppressed by the fourth generation mixing angles. The Yukawa g_D in $SU(4)_F$ is $m_{b'}/m_b$ times g_D in $SU(3)_F$. The fourth generation charge -1/3 quark has a mass greater than 22 GeV[11], so g_D is some factor of 5 or more larger in $SU(4)_F$. All of the heavy scalar boson masses are therefore bounded by numbers a factor of 5 larger than those quoted above.

There are also constraints arising from limits on $B^0 - \bar{B}^0$ and $D^0 - \bar{D}^0$ mixing. The limits from $B^0 - \bar{B}^0$ are not as severe as from $K^0 - \bar{K}^0$ because of the poorer experimental limits and the small splittings among the heavy h scalars. Simply due to the relative precision of the experimental measurements, the bounds from $D^0 - \bar{D}^0$ mixing are not as severe unless $g_U/g_D \gtrsim 10$.

Within our framework the charged leptons cannot be incorporated realistically into the scheme with only two weak Higgs doublet representations. Depending on whether lepton doublets transform as the fundamental or conjugate fundamental representations of the horizontal group, we are led to two different, but both wrong, sets of mass relations. In one case, $m_d/m_s = m_e/m_\mu$ and $m_s/m_b = m_\mu/m_\tau$. For the other, $m_u/m_c = m_e/m_\mu$ and $m_c/m_t = m_\mu/m_\tau$ are obtained. This second case, analysed by Glashow[12], predicts a top quark mass on the order of 20 GeV.

Spectator fermions, $SU(3) \times SU(2) \times U(1)$ singlets with family symmetry quantum numbers, are a necessity for our models. Low energy experiments, notably $K \rightarrow \pi + nothing$, constrain their effective couplings with ordinary matter. As interactions between spectators and quarks must involve family gauge bosons, spectator contributions to rare processes are adequately suppressed. Cosmological considerations provide more general restrictions. The lightest spectator, which is stable, should not contribute too much to the present energy density of the universe. The long-lived ($\tau \gtrsim 1$ sec) and stable spectator fermion energy density must be sufficiently low at the time of nucleosynthesis to avoid dramatically changing the expansion rate of the universe; however, the decoupling temperature for the spectators is sufficiently high that this should not be a serious problem. The precise spectator content is dictated, in part, by the lepton family representations. In view of the considerable freedom in the choice of representations of spectators, an analysis of an example will be relegated to ref.[4].

The motivation of this effort has been to examine how the successive breaking of a nonabelian family symmetry leads to hierarchies in quark masses. Within our framework, it is necessary to add a fourth generation to generate fermion mass hierarchies without the input of small tree level parameters. In the $SU(4)_F$ theory, the fermion mass ratios among the first three generations scale as the fourth power of ratios of vev's breaking the family group. A rather small hierarchy in the breaking of $SU(4)_F$ leads to acceptable fermion mass ratios.

The quark mass matrices depended on too many parameters for predictive relations between mixing angles and mass ratios. It is, in general, quite difficult to avoid this situation without imposing zeroes in the quark mass matrices. Our models reproduce the qualitative feature that the K-M angles fall off away from the diagonal. A novel expression for θ_{Cab} is suggested, however, the small size of the third generation mixing angles is not explained.

The versions of the nonabelian family models presented here have unsuccessful relations between ratios of quark and lepton masses with the most natural family representation assignments. Furthermore, the number of additional fields required is unsatisfying. Nevertheless, our successes suggest that the general idea of using a nonabelian symmetry to generate realistic Yukawa couplings from a minimal set of fundamental Yukawa's is worthy of further study.

Acknowledgements

We thank S. Dimopoulos, H. Georgi, K. Lane and M. Peskin for interesting conversations. M. S. thanks the Aspen Center for their hospitality.

References

[1] S. Weinberg, *Phys. Rev.* **D13** (1976) 974 and **D19** (1979) 1277;
L. Susskind, *Phys. Rev.* **D20** (1979) 2619.

[2] S. Dimopoulos and L. Susskind, *Nucl. Phys.* **B155** (1979) 237;
E. Eichten and K. Lane, *Phys. Lett.* **90B** (1980) 125.

[3] An incomplete list of work on nonabelian family groups in this spirit, gauged or otherwise, is
S.Weinberg, *Phys. Rev.* **D5** (1972) 1962;
H. Georgi and S. Glashow, *Phys. Rev.* **D6** (1972) 2977 and **D7** (1973) 2457;
F. Wilczek and A. Zee, *Phys. Rev. Lett.* **42** (1979) 421;
C. L. Ong, *Phys. Rev.* **D19** (1979) 2738;
D. Jones, G. Kane and J. Leveille, *Nucl. Phys.* **B198** (1982) 45;
T. Kuo and N. Nakagawa, *Nucl. Phys.* **B250** (1985) 641;
P. Ramond, University of Florida preprint UFTP-85-3, 1985;
W.-S. Hou and A. Soni, *Phys. Rev. Lett.* **54** (1985) 2083.
See also Ref. [5].

[4] M. Soldate, in preparation, FERMILAB-Pub-86/87-T, 1986.

[5] M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, ed. by P. Van Nieuwenhuizen and D. Z. Freedman, (North-Holland, 1979), p. 315;
M. Bowick and P. Ramond, *Phys. Lett.* **103B** (1981) 338;
F. Wilczek, Erice lectures, 1983 ;
G. Gelmini, J.-M. Gérard, T. Yanagida and G. Zoupanos, *Phys. Lett.* **135B** (1984) 103;
K. Kang and M. Shin, Brown University preprint, BROWN HET-570, 1985.

[6] R. Cahn and H. Harari, *Nucl. Phys.* **B176** (1980) 135.

[7] See F. Wilczek, *Phys. Rev. Lett.* **49** (1982) 1549 for analogous comments in an SU(2) model.

[8] Early references are R. Gatto, G. Sartori and M. Tonin, *Phys. Lett.* **28B** (1968) 128;
N. Cabibbo and L. Maiani, *Phys. Lett.* **28B** (1968) 131.
A more recent reference is H. Fritzsch, *Nucl. Phys.* **B155** (1979) 189.
A more complete list may be found in Wilczek and Zee, Ref. [3].

[9] J. Trampetić, *Phys. Rev.* **D27** (1983) 1565.

[10] S. Glashow and S. Weinberg, *Phys. Rev.* **D15** (1977) 1958; E. Paschos, *Phys. Rev.* **D15** (1977) 1966.

[11] For bounds on new quarks at PETRA, see for example, B. Adeva, *et al.*, *Phys. Lett.* **152B** (1985) 439.

[12] S. Glashow, *Phys. Rev. Lett.* **45** (1980) 1914.